Exercises

4.1 Verify the schedulability and construct the schedule according to the RM algorithm for the following set of periodic tasks:

	C_i	T_i
τ_1	2	6
τ_2	2	8
τ_3	2	12

4.2 Verify the schedulability and construct the schedule according to the RM algorithm for the following set of periodic tasks:

	C_i	T_i
τ_1	3	5
τ_2	1	8
τ_3	1	10

4.3 Verify the schedulability and construct the schedule according to the RM algorithm for the following set of periodic tasks:

	C_i	T_i
τ_1	1	4
τ_2	2	6
τ_3	3	10

4.4 Verify the schedulability under RM of the following task set:

	C_i	T_i
τ_1	1	4
τ_2	2	6
τ_3	3	8

4.5 Verify the schedulability under EDF of the task set shown in Exercise 4.4, and then construct the corresponding schedule.

4.6 Verify the schedulability under EDF and construct the schedule of the following task set:

	C_i	D_i	T_i
τ_1	2	5	6
τ_2	2	4	8
τ_3	4	8	12

4.7 Verify the schedulability of the task set described in Exercise 4.6 using the Deadline-Monotonic algorithm. Then construct the schedule.

SOLUTIONS FOR CHAPTER 4

4.1 The processor utilization factor of the task set is

$$U = \frac{2}{6} + \frac{2}{8} + \frac{2}{12} = 0.75$$

and considering that for three tasks the utilization least upper bound is

$$U_{lub}(3) = 3(2^{1/3} - 1) \simeq 0.78$$

from the Liu and Layland test, since $U \leq U_{lub}$, we can conclude that the task set is schedulable by RM, as shown in Figure 1.4.

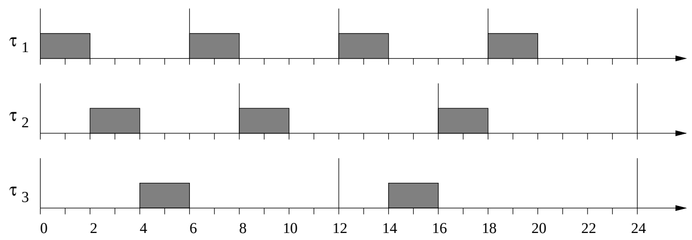

Figure 1.4 Schedule produced by Rate Monotonic for the task set of Exercise 4.1.

4.2 The processor utilization factor of the task set is

$$U = \frac{3}{5} + \frac{1}{8} + \frac{1}{10} = 0.825$$

which is greater than $U_{lub}(3)$. Hence, we cannot verify the feasibility with the Liu and Layland test. Using the Hyperbolic Bound, we have that:

$$\prod_{i=1}^{n}(U_i + 1) = 1.98$$

which is less than 2. Hence, we can conclude that the task set is schedulable by RM, as shown in Figure 1.5.

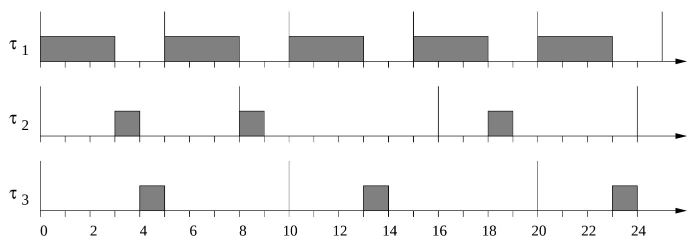

Figure 1.5 Schedule produced by Rate Monotonic for the task set of Exercise 4.2.

4.3 Applying the Liu and Layland test we have that

$$U = \frac{1}{4} + \frac{2}{6} + \frac{3}{10} = 0.88 > 0.78$$

so we cannot say anything. With the Hyperbolic Bound we have that

$$\prod_{i=1}^{n}(U_i + 1) = 2.16 > 2$$

so we cannot say anything. Applying the Response Time Analysis we have to compute the response times and verify that they are less than or equal to the relative deadlines (which in this case are equal to periods). Hence, we have:

$$R_1 = C_1 = 1$$

So τ_1 does not miss its deadline. For τ_2 we have:

$$\begin{aligned} R_2^{(0)} &= \sum_{j=1}^{2} C_j = C_1 + C_2 = 3 \\ R_2^{(1)} &= C_2 + \left\lceil \frac{R_2^{(0)}}{T_1} \right\rceil C_1 = 2 + \left\lceil \frac{3}{4} \right\rceil 1 = 3 \end{aligned}$$

So $R_2 = 3$, meaning that τ_2 does not miss its deadline. For τ_3 we have:

$$\begin{aligned} R_3^{(0)} &= \sum_{j=1}^{3} C_j = C_1 + C_2 + C_3 = 6 \\ R_3^{(1)} &= C_3 + \left\lceil \frac{R_3^{(0)}}{T_1} \right\rceil C_1 + \left\lceil \frac{R_3^{(0)}}{T_2} \right\rceil C_2 = 3 + \left\lceil \frac{6}{4} \right\rceil 1 + \left\lceil \frac{6}{6} \right\rceil 2 = 7 \\ R_3^{(2)} &= 3 + \left\lceil \frac{7}{4} \right\rceil 1 + \left\lceil \frac{7}{6} \right\rceil 2 = 9 \\ R_3^{(3)} &= 3 + \left\lceil \frac{9}{4} \right\rceil 1 + \left\lceil \frac{9}{6} \right\rceil 2 = 10 \\ R_3^{(4)} &= 3 + \left\lceil \frac{10}{4} \right\rceil 1 + \left\lceil \frac{10}{6} \right\rceil 2 = 10 \end{aligned}$$

So $R_3 = 10$, meaning that τ_3 does not miss its deadline. Hence we can conclude that the task set is schedulable by RM, as shown in Figure 1.6.

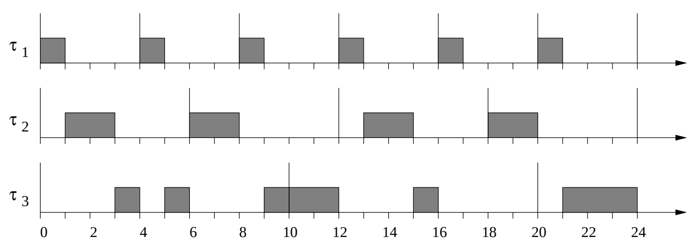

Figure 1.6 Schedule produced by Rate Monotonic for the task set of Exercise 4.3.

4.4 Applying the Response Time Analysis, we can easily verify that $R_3 = 10$ (see the solution of the previous exercise), hence the task set in not schedulable by RM.

4.5 Since

$$U = \frac{1}{4} + \frac{2}{6} + \frac{3}{8} = 0.96 < 1$$

the task set is schedulable by EDF, as shown in Figure 1.7.

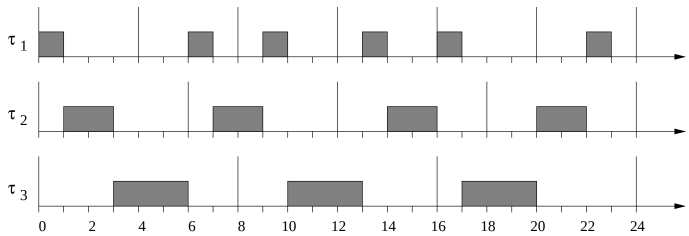

Figure 1.7 Schedule produced by EDF for the task set of Exercise 4.5.

4.6 Applying the processor demand criterion, we have to verify that

$$\forall L \in \mathcal{D} \quad \sum_{i=1}^{n} \left\lfloor \frac{L + T_i - D_i}{T_i} \right\rfloor C_i \ \leq \ L.$$

where

$$\mathcal{D} = \{d_k \mid d_k \leq \min(L^*, H)\}.$$

For the specific example, we have

$$\begin{aligned} U &= \frac{2}{6} + \frac{2}{8} + \frac{4}{12} = \frac{11}{12} \\ L^* &= \frac{\sum_{i=1}^{n}(T_i - D_i)U_i}{1 - U} = 32 \\ H &= \text{lcm}(6, 8, 12) = 24. \end{aligned}$$

Hence, the set of checking points is given by $\mathcal{D} = \{4, 5, 8, 11, 12, 17, 20, 23\}$. Since the demand in these intervals is $\{2, 4, 8, 10, 12, 14, 20, 22\}$ we can conclude that the task set is schedulable by EDF. The resulting schedule is shown in Figure 1.8.

4.7 Applying the Response Time Analysis, we have to start by computing the response time of task τ_2, which is the one with the shortest relative deadline, and hence the highest priority:

$$R_2 \ = \ C_2 = 2.$$

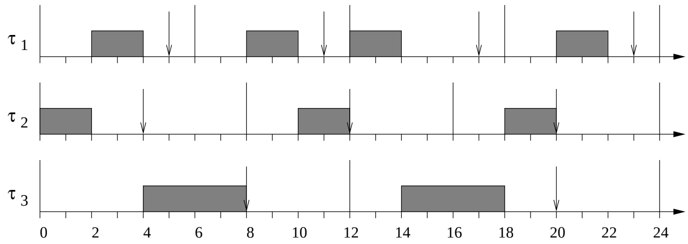

Figure 1.8 Schedule produced by EDF for the task set of Exercise 4.6.

So τ_2 does not miss its deadline. For τ_1 we have:

$$R_1^{(0)} = \sum_{j=1}^{2} C_j = C_1 + C_2 = 4$$

$$R_1^{(1)} = C_1 + \left\lceil \frac{R_1^{(0)}}{T_2} \right\rceil C_2 = 2 + \left\lceil \frac{4}{8} \right\rceil 2 = 4$$

So $R_1 = 4$, meaning that τ_1 does not miss its deadline. For τ_3 we have:

$$R_3^{(0)} = \sum_{j=1}^{3} C_j = C_1 + C_2 + C_3 = 8$$

$$R_3^{(1)} = C_3 + \left\lceil \frac{R_3^{(0)}}{T_2} \right\rceil C_2 + \left\lceil \frac{R_3^{(0)}}{T_1} \right\rceil C_1 = 4 + \left\lceil \frac{8}{8} \right\rceil 2 + \left\lceil \frac{8}{6} \right\rceil 2 = 10$$

And since $R_3^{(1)} > D_3$, we can conclude that the task set is not schedulable by DM. The resulting schedule is shown in Figure 1.9.

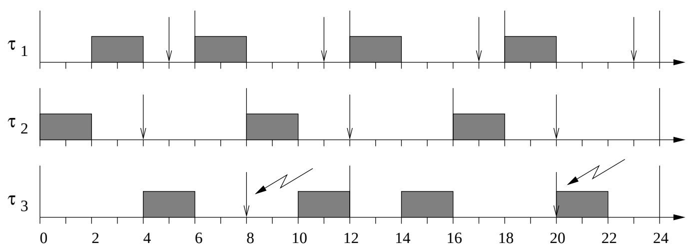

Figure 1.9 Schedule produced by Deadline Monotonic for the task set of Exercise 4.7.